SOFTWARE "SIRIUS-C" FOR SYNTHESIS OF OPTIMAL CONTROL BY QUEUES

Alexander N. Dudin, Valentina I. Klimenok, Gennadiy V. Tsarenkov,
Olga V. Semenova, Alexander A. Birukov
Laboratory of Applied Probabilistic Analysis
Department of Applied Mathematics and Computer Sciences
Belarus State University
4, F.Skorina Ave., Minsk-50, 220050, Belarus
dudin@bsu.by
klimenok@bsu.by
g.tsarenkov@sam-solutions.net
semenovaov@bsu.by
al_birukov@mail333.com

KEYWORDS
Queue, Batch Markovian Arrival Process, Control, Software

ABSTRACT
Software "SIRIUS-C" is created for the synthesis of optimal parametric strategies of control by the single-server queues having several possible operation modes. Operation modes differ by the intensity of customer arrival, service, retrials and cost of utilization. Modes can be changed at the service completion or customers arrival epochs depending on the current queue length. Software, which is described in this paper, provides the optimal values of parameters of control strategy and the optimal value of the cost criterion under the given descriptors of the operation modes and cost coefficients.

INTRODUCTION

Optimal dynamic control by the queueing system operation is the highest level problem among all other problems of the queueing theory (static optimization, capacity planning, performance evaluation, etc.). However, this problem is still far from successful solution. The presented software makes a step in solving this problem. This software is the extension of software "SIRIUS+" (Dudin et al. 2000) and "SIRIUS++" (Dudin et al. 2002). The former one contains the modules which allow to calculate queue length distribution and some performance characteristics for variety of queueing systems having the BMAP (Batch Markovian Arrival Process) input. The later one includes also a lot of modules devoted to performance evaluation of retrial queues. Semi-Markovian (SM) service is allowed in (Dudin et al. 2002) in opposite to (Dudin et al. 2000) where only recurrent service process was dealt with.

The paper contains description of the software "SIRIUS-C" and its functional modules as well as the example of calculations.

GENERAL DESCRIPTION OF FUNCTIONAL MODULES

The current version of "SIRIUS-C" consists of 12 functional modules which can be divided into four groups. Two program modules ($\boldsymbol{BMAP/G(j)/1}$ and $\boldsymbol{BMAP/G(j,k)/1}$) are contained, in the package "SIRIUS++" but they essentially deal with the controlled queues of the $BMAP/G/1$ type with the threshold and hysteresis strategy of control.

Optimal Admission Control

The modules of the first group deal with the controlled queues of the $GI/PH/1$ type having $N, N \geq 2$, operation modes. Under the fixed rth operation mode, the inter-arrival times are described by distribution function $A_r(t)$ and intensity $\lambda_r = \int_0^\infty (1 - A_r(t))dt$. The service time is characterized by the PH distribution with irreducible representation (β, S) and mean service time $b_1 = \beta(-S)^{-1}\mathbf{e}$. The cost of operation in this mode per unit time is $c_r, r = \overline{1,N}$. Denote the traffic intensity (load) in the rth mode by $\rho_r = \lambda_r b_1, r = \overline{1,N}$.

Without the loss of generality, we assume that $\rho_1 \geq \cdots \geq \rho_N$. Mention that the decreasing of arrival intensity can be obtained, e.g., by means of rejection of some part of customers. So, the model can be called as the model for the optimal admission policy determination.

Corresponding to the loads enumeration, we assume that the costs of the modes satisfy inequalities $0 \leq c_1 \leq \cdots \leq c_N$. So, the first operation mode admits the maximal input flow into the system and is less expensive. The Nth operation mode assumes the minimal intensity of accepted customers and the maximal charge. Thus, it makes sense to use modes with small numbers when the queue length in the system is small and the modes having the higher numbers should be exploited when the queue length increases.

To evaluate the quality of the system operation, the following criterion is used:

$$I = aW + \sum_{r=1}^{N} c_r P_r, \tag{1}$$

where W is the average waiting time, a is the penalty for a customer waiting during unit of time, P_r is the average fraction of using the rth mode, $r = \overline{1,N}$.

It is assumed that a decision-maker observes the queue length in the system at customers arrival epochs and has an opportunity to change the operation modes at each such an epoch depending on the current length of the queue.

More detailed description of the model and the results of its investigation can be found in (Dudin and Klimenok 2003). Following to this paper as well to the traditions in optimal control by the queues, see, e. g. (Tijms 1976), the optimal strategy is looked for in the class of the so called multithreshold strategies. Such a strategy is defined by a set of integers $j_1, \ldots, j_{N-1}$ called the thresholds. It is assumed that $-1 = j_0 < j_1 \leq j_2 \leq \cdots \leq j_{N-1} < j_N = \infty$. If a queue length i at a given decision making epoch satisfies the inequality $j_{r-1} < i \leq j_r$, then the system will operate in the rth mode, $r = \overline{1,N}$.

Such a simple structure of control strategy allows to investigate the queue behaviour under the fixed strategy. Stationary queue length distribution at arbitrary time and decision making epochs as well as the waiting time distribution are calculated in (Dudin and Klimenok 2003).

The described first block of program modules consists of three modules. The first one ($\boldsymbol{G(N)/PH/1}$) calculates the optimal set $(j_1^*, \ldots, j_{N-1}^*)$ of the thresholds for any fixed set of distributions $A_r(t)$, cost coefficients $a, c_r, r = \overline{1,N}$, and service time distribution parameters (β, S). The optimal value of the criterion (1) and its components are calculated as well.

The second module $\boldsymbol{G(2)/PH(j)/1}$ realizes a simpler algorithm for the case $N = 2$.

The third model $\boldsymbol{G(2)/PH/1}$ (***BMAP*** **negative arrivals**) is the generalization of the first one to the case when there is an additional arriving flow of negative customers. This flow is described by the BMAP. Arrival of a batch of negative customers causes immediate departure of corresponding number of customers from the head of the system. The cost criterion includes the additional additive term dH, where H is the average number of customers leaving the system per unit of time due to the negative arrivals, d is the penalty for one customer loss. The theoretical background for this program is given in (Kim et al. 2004a).

Optimal Service Rate Control

The modules of the second group are devoted to the controlled queues of the $BMAP/SM/1$ type. The queue has N possible operation modes, $N \geq 2$. Under the fixed rth operation mode, the matrices, which define transitions of the $BMAP$ underlying process, depend on r. The semi-Markovian kernel, which defines the service, also depends on $r, r = \overline{1,N}$.

It is also assumed that the loads ρ_r and the costs c_r of modes utilization are enumerated as
$\rho_1 \geq \cdots \geq \rho_N$, $c_1 \leq \cdots \leq c_N$.

In contrast to the previous program block, here it is assumed that the decision making epochs are the service completions.

Quality of the system operation is evaluated by criterion

$$I = a\tau^{-1}L + \sum_{r=1}^{N} c_r P_r, \tag{2}$$

where the quantities P_r and cost coefficients c_r have the same sense as in criterion (1), L is the average queue length at the service completion epochs, τ is average time between such epochs, a is a holding cost (charge for one customer staying in the queue per unit of time).

The optimal control policy is also found in the class of parametric strategies.

The first module of this block ($\boldsymbol{BMAP/SM(j)/1}$) calculates the optimal thresholds $(j_1^*, \ldots, j_{N-1}^*)$ of multithreshold strategy, the optimal value of the criterion (2) and values of its components. Stationary distribution of the queue under the fixed set of thresholds $(j_1^*, \ldots, j_{N-1}^*)$ is calculated and is available for a user. Theoretical background for creating this module consists of the result of (Dudin 1998) generalized to the case of semi-Markovian service process.

The second module of this group (***BMAP/SM(j)/1*** **(retrials)**) calculates the optimal multithreshold strategy of control for the model distinguishing from the first model of this block by the following assumption. In opposite to the first model, which has an infinite buffer, we assume that the system has no buffer at all. Arriving customers go to the so called orbit if they meet a busy server upon arrival. They make attempts to get the service from the orbit. The total flow of these attempts (retrials) from the orbit has the intensity $\alpha_i^{(r)} = i\alpha^{(r)} + \gamma^{(r)}$ when the system operates in the rth mode and i customers present in the orbit, $\alpha^{(r)} \geq 0, \gamma^{(r)} \geq 0, i > 0, r = \overline{1, N}$. The cost criterion coincides with (2).

It is assumed that the decision maker observes the number of customers on the orbit at the service completion epochs. This assumption is realistic in a lot of local area computer networks or internet access points. The theoretical background of this module and some results of its work are given in (Kim et al. 2004b). The investigated model is very general because all the processes (arrival, service, retrial) are assumed to be controlled. Sure, it can be applied for less general real models. E.g., the problem of optimal retrial policy calculation can be solved. Note, that it is possible to use centralized retrial policy when $\alpha_i^{(r)} = \gamma^{(r)}$ (only one customer is allowed to retry or the individual retrial intensity is inversely proportional to the number of customers on the orbit) alternating with decentralized one when $\alpha_i^{(r)} = i\alpha^{(r)}$.

Optimal Service Rate Control For The System With Exhaustive Group Service Modes

The third block of functional modules also deals with the problem of optimal service rate control in the queues of the $BMAP/SM/1$ type. The main difference from the modules of the previous block is the following. In all available modes of system operation, the customers are assumed to be served one-by-one in the previous block. In the present block, it is assumed that the customers can be served in groups of varying size in some or all operation modes.

The first module of this block (***BMAP/SM(j)/1(G)/K*** **with group service**) deals with the $BMAP/SM/1/K$ queue having N available operation modes. The system has a finite buffer of capacity $K, K \geq 1$. The discipline of customers admission is partial admission. It means that the customers in arriving batch are admitted to the extent of available buffer and all others are considered lost. The system operation is described as follows. The thresholds $L, j_1, \ldots, j_{N-1}$ are fixed, $L - 1 = j_0 < j_1 \leq \cdots \leq j_{N-1} \leq j_N = K$. If the queue length i at a service completion epoch is less than L, no service is offered until the queue builds up to L or more. After that all customers presenting in the system are served in the first mode. If the queue length i at a service completion epoch satisfies the inequality $j_{r-1} < i \leq j_r$, then the entire group of i customers is served in the rth mode, $r = \overline{1, N}$. Service time of the groups in the rth mode is defined be the semi-Markovian kernel $B^{(r)}(t), r = \overline{1, N}$.

The systems with such a group service were intensively investigated by S. Chakravarthy, see, e.g., reference list in (Dudin and Chakravarthy 2003).

The cost criterion has the form

$$I = aL + \sum_{r=0}^{N} c_r P_r + d\lambda Q_{reject}, \tag{3}$$

where L is the average queue length at arbitrary time, $P_r, r \geq 1$, have the same sense as above, P_0 is an average fraction of time when the server is idle, Q_{reject} is a probability of an arbitrary customer loss, λ is the fundamental rate of the BMAP, $a, c_r, r = \overline{0, N}, d$ are the corresponding cost coefficients.

The program module calculates the optimal set $(L^*, j_1^*, \ldots, j_{N-1}^*)$ of the thresholds as well as the optimal value of criterion (3) and its components. The theoretical background of this module and extensive numerical illustration of its work are contained in (Dudin and Chakravarthy 2003).

The second module of this program block (***BMAP/SM(j)/1(G)***) deals with the queueing system, which also has N available modes. First $N_1, N_1 < N$, modes assume the service of customers one-by-one. The rest of modes assume group service as described above. The module calculates the optimal set $(j_1^*, \ldots, j_{N-1}^*)$ of thresholds and the optimal value of criterion (3). The theoretical background for this module is presented in (Birukov 2004).

The third module of this block (***BMAP/G(j,k)/1(G)***) is devoted to the system, which has only two available modes. The customers are served one-by-one in the first mode and in groups of varying size in the second mode. The cost criterion (2) is assumed to be supplemented by the term gM, where M is the average number of modes switching per unit of time, $g, g \geq 0$, is the penalty for one switch. The strategy of control is hysteresis one.

The presence of the charge for modes switch causes, in general, non-optimality of the threshold strategy of control. So, the optimal policy is found in the class of hysteresis strategies. The hysteresis strategy is defined by two thresholds, j and k, $0 \leq j \leq k < +\infty$. If a queue length i at a given decision making epoch satisfies inequality $i \leq j$, the system will operate in the first mode. If $i > k$ then the system will operate in the second mode. If $j < i \leq k$ then the system maintains the current operation mode.

The discussed module of software calculates the optimal set (j^*, k^*) of the thresholds, the optimal value of the supplemented criterion (2) and its components. The theoretical background of this module for the case of service customers one-by-one in both modes is given in (Dudin and Nishimura 1999a) and (Dudin and Nishimura 2000). The theoretical background of this module for a present case and the results of some numerical experiments are presented in (Dudin and Chakravarthy 2002).

Optimal Service Rate Control For The Systems With Disasters

This block of functional modules concerns the queues of the $BMAP/SM/1$ type. It is supposed that the additional MAP flow of disasters arrives into the system. The disaster arrival causes the instant removal of all customers (including the one on the server) from the system. Such models with instantaneous recovering in the case of no control are investigated in (Dudin and Nishimura 1999b), (Dudin and Semenova 2004). The model when the server is recovering after disaster occurrence during the random time having a known distribution function is considered in (Dudin and Karolik 2001).

This block contains four modules. All modules deal with the system having two available modes. The cost criterion has a form (2) supplemented by additional term hR, where R is the average number of customers lost per time unit (due to a disaster occurrence), h is a charge for one customer loss. The strategy of control is the threshold one. We distinguish here a classic threshold strategy and a modified threshold strategy. A difference between these strategies is the following. If the queue is idle at a given customers departure epoch, the next customer is assumed to be served in the first mode according to a classic strategy. The modified strategy assumes that the operation mode will be defined in this situation at the next service beginning epoch. If the number of customers at this epoch is less than the fixed threshold j, the system will operate in the first mode. Otherwise, the second mode is selected.

Two of modules of this block
(***BMAP/SM*(2)/1(MAP disasters**
and ***BMAP/SM*(2)/1(MAP disasters and recovery**)
calculate the optimal threshold for a classic strategy. Two others
***BMAP/SM*(2*m*)/1** (**MAP disasters**) and
***BMAP/SM*(2*m*)/1(MAP disasters and recovery**) deal with a modified strategy. One module of each group assumes instantaneous recovering of the server, the other accounts the random recovery time. The theoretical background and some numerical examples are presented in papers (Semenova 2003) and (Semenova 2004).

"SIRIUS-C" SOFTWARE PACKAGE

The modules of "SIRIUS-C" software are written in C++ language using Microsoft Visual Studio. The facilitating environment has been developed using Borland Delphi 7. User interface is multi-lingual, currently supports English and Russian languages. All visual components of the user interface are well-known to the Windows user, so there is no particular training is needed to work with the software. Of cause, the user should be familiar with queueing theory, at least definitions of BMAP-flows and semi-Markovian processes and their subsequent applications.

Additionally, the source code of modules is compatible with GNU C++ compiler version 3.2 and therefore can be built on a number of non-Windows platforms.

The set of available distributions in functional modules coincides with the one described in (Dudin et al. 2000). In some modules, this set is supplemented by the lognormal and Weibull distributions. The way of the BMAP's parameters determination is also the same as in (Dudin et al. 2000).

The look-and-feel of the software is given below in "Numerical Results" section.

Requirements to the hardware can vary depending on tasks that the software package will be used for. For studying purposes it is enough to have Pentium processor, 350 MHz with 128MB of memory. However, for the serious engineering calculation it may require to have 512MB of memory, in order to be able to evaluate the characteristics of the queues with BMAP state space dimension about 15 and semi-Markovian process state space dimension about 5. There are several limitations that currently holds in the software, so the dimension of the BMAP state space should not exceed 20 and dimension of the semi-Markovian service should not exceed 15. Generally speaking we have positive results for calculating the queues where transition probabilities matrices have dimension about several hundreds.

Calculation time depends on the selected model, system load and other parameters and may vary from milliseconds up to several hours.

NUMERICAL RESULTS

To illustrate the work of one of the discussed modules we present some numerical results.

Let us consider $GI/PH/1$ queueing system having five available input modes. In the first four modes, the inter-arrival time has hypererlangian distribution

$$A_r(t) = \sum_{i=1}^{k} q_i \int_0^t \frac{\gamma_i^{(r)}(\gamma_i^{(r)}\tau)^{h_i^{(r)}-1}}{(h_i^{(r)}-1)!} e^{-\gamma_i^{(r)}\tau} d\tau, \quad r = \overline{1,4},$$

where $k = 2, q_1 = 0,4,\ q_2 = 0,6,\ h_1^{(1)} = 1,\ h_2^{(1)} = h_1^{(2)} = 2,\ h_2^{(2)} = h_1^{(3)} = 3,\ h_2^{(3)} = h_1^{(4)} = 4,\ h_2^{(4)} = 5,$ $\gamma_1^{(1)} = 10,\ \gamma_2^{(1)} = \gamma_1^{(3)} = \gamma_2^{(3)} = \gamma_2^{(4)} = 20,\ \gamma_1^{(2)} = 14,\ \gamma_2^{(2)} = 24,\ \gamma_1^{(4)} = 25.$

The inter-arrival times in the fifth mode are deterministic and equal to 0.3. The mean intensities λ_r of the customers input in the rth mode are the following: $\lambda_1 = 10$, $\lambda_2 = 7.74$, $\lambda_3 = 5.55$, $\lambda_4 = 4.67$, $\lambda_5 = 3.33$.

Screen shot of the input data for this model is given in Fig. 2.

The customer service is characterized by the vector $\beta = (0.4 \quad 0.6)$ and the matrix $S = \begin{pmatrix} -5 & 1 \\ 2 & -8 \end{pmatrix}$. The mean service time $b_1 = 0.205$.

The cost criterion has the form (1), the cost coefficients are the following: $a = 70, c_1 = 100, c_2 = 150, c_3 = 300, c_4 = 370, c_5 = 550$.

Denote by I_r the value of the cost criterion (1) when only one fixed rth mode is exploited, $r = \overline{1,5}$. The values $I_r, r = \overline{1,3}$, are infinite because the input intensity exceeds the service rate in modes 1, 2, 3. The rest of values I_r are calculated as $I_4 = 574.36$, $I_5 = 562.67$. So, if there is no possibility to control the operation mode, the system should choose the 5th operation mode.

As the result of calculation, we get the following. The optimal set of the thresholds is $j_1 = -1, j_2 = j_3 = 1, j_4 = 5$ (see Fig. 3).

It means that only three among five available operation modes (modes number 2, 4 and 5) are included into the optimal set of exploited modes. The optimal value I^* of the cost criterion is equal to 441.08. It is evident that I^* gives the relative profit 21% comparing to exploiting only the 5th mode.

It illustrates the necessity of the input control and possibility to reduce the cost of the system operation by means of this control.

Table 1 contains the values of the optimal cost criterion for the fixed different combinations of available modes.

Table 1: The Values of the Optimal Cost Criterion for the Fixed Combination of Operation Modes

Set of modes	Optimal thresholds	Optimal value of the cost criterion
1	–	∞
2	–	∞
3	–	∞
4	–	574.36
5	–	562.67
1,4	0	574.22
1,5	1	494.90
2,4	0	571.72
2,5	2	461.19
3,4	0	572.70
3,5	3	454.15
4,5	5	453.22
1,2,4	0,0	574.22
1,2,5	0,2	465.65
1,3,5	0,3	454.63
1,3,4	0,0	574.22
1,4,5	0,5	448.23
2,3,4	0,0	571.72
2,3,5	0,3	450.16
2,4,5	1,5	441.08
1,2,3,4	0,0,0	574.22
1,2,3,5	0,1,3	453.94
1,2,4,5	0,1,5	444.08
2,3,4,5	1,1,5	441.09
1,3,4,5	0,1,5	445.61
1,2,3,4,5	0,1,1,5	444.31

Dependence of the cost criterion (1) on two thresholds when the optimal set (2,4,5) of the modes is exploited is given on Fig. 1.

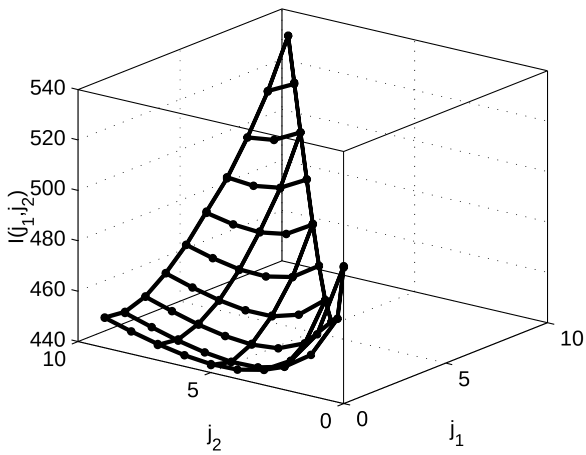

Figure 1: The Dependence of the Cost Criterion Value on the Thresholds

Probabilities to see different number of customers in the system at the customer arrival epochs under the optimal control policy are given in the table 2.

Table 2: The Stationary Probabilities under the Optimal Control Policy

Number of customers i	Probability π_i
0	0.08393
1	0.12654
2	0.16456
3	0.15399
4	0.14516
5	0.13222
6	0.10203
7	0.04796
8	0.02280
9	0.01086
10	0.00517
11	0.00246
12	0.00117
$\sum_{i=0}^{12} \pi_i$	0.99893

Note, that the program module is able to provide a user by separate values of all components of the cost criterion (1) for any set of the thresholds. It potentially allows to user to use some program tools like EXCEL and so on to calculate the optimal value of the cost criterion for any fixed set of the cost coefficients without further calculation by means of "SIRIUC-C". It allows to the user, e.g., to generate the optimal in some sense set of the cost coefficients (tariffs) for the real life system modelled by a given module.

CONCLUSION

The content and functions of the modules of "SIRIUS-C" software are described in brief. The modules allow effectively solve the problem of constructing the optimal threshold or hysteresis strategy when the real-life system has several different operation modes and possibility of changing the modes in real time depending on the current number of customers in the system. The examples of such systems are: resource of telecommunication network, which handles a mixture of flows having different importance for the system or different requirements to the response time; systems with access on demand which have a part of effective channel bandwidth reserved for dynamic redistribution among the customers upon request; systems, which combine transmission of several types of information having different requirements to the quality of service and having a possibility to distribute the bandwidth dynamically, etc.

REFERENCES

Birukov, A. 2004. "Optimal multi-threshold control of the BMAP/SM/1 system with single/group service modes". *International Journal of Production Research* (submitted).

Dudin, A.N. 1998. "Optimal multi-threshold control for a BMAP/G/1 queue with N service modes". *Queueing Systems* 30, No.3-4, 273-287.

Dudin, A.N. 2002. "Optimal hysteresis control for an unreliable BMAP/SM/1 system with two operation modes". *Automation and Remote Control* 63, No.10, 1585-1596.

Dudin, A.N. and S. Chakravarthy. 2002. "Optimal hysteretic control for the BMAP/G/1 system with single and group service modes". *Annals of Operations Research* 111, 153-169.

Dudin, A.N. and S. Chakravarthy. 2003. "Multi-threshold control for the BMAP/SM/1/K queue with group services". *Journal of Applied Mathematics and Stochastic Analysis*, 16, No.4, 327-348.

Dudin, A.N. and A.V. Karolik. 2001. "BMAP/SM/1 queue with Markovian input of disasters and non-instantaneous recovery". *Performance Evaluation* 45, No.1, 19-32.

Dudin, A.N. and V.I. Klimenok. 2003. "Optimal admission control in a queueing system with heterogeneous traffic". *Operations Research Letters* 28, No.4, 108-118.

Dudin, A.N.; V.I. Klimenok; I.A. Klimenok; V.V. Borokhovsky; A.V.Karolik and G.V. Tsarenkov. 2000. "Software "SIRIUS+" for evaluation and optimization of queues with the BMAP-input". In *Advances in Algorithmic Methods for Stochastic Models*, eds. G. Latouche and P. Taylor. Notable Publications, N.J., 114-133.

Dudin, A.N.; V.I. Klimenok and G.V. Tsarenkov. 2002. "Software "SIRIUS++" for performance evaluation of modern communication networks". *Modelling and Simulation 2002. 16th European Simulation Multi-conference.* Darmstadt, 489-493.

Dudin, A.N. and S. Nishimura. 1999a. "Optimal control for a BMAP/G/1 queue with two service modes". *Mathematical Problems in Engineering* 5, No.3, 255-273.

Dudin, A.N. and S. Nishimura. 1999b. "A BMAP/SM/1 queueing system with Markovian arrival of disasters". *Journal of Applied Probability* 36, No.3, 868-881.

Dudin, A.N. and S. Nishimura. 2000. "Optimal hysteretic control for a BMAP/SM/1/N queue with two operation modes". *Mathematical Problems in Engineering* 5, No.5, 397-420.

Dudin, A. and O. Semenova. 2004. "Stable algorithm for stationary distribution calculation for a BMAP/SM/1 queueing system with Markovian arrival input of disasters". *Journal of Applied Probability* 41, No.2.

Kim, C.S.; O.V. Semenova and A.N. Dudin. 2004a. "Optimal multithreshold input control for GI/PH/1 queueing system with BMAP input of negative arrivals". *Automation and Remote Control* (submitted).

Kim, C.S.; V.I. Klimenok; A.V. Birukov and A.N. Dudin. 2004b. "Optimal multi-threshold control by the BMAP/SM/1 retrial system". *Annals of Operations Research* (submitted).

Semenova, O.V. 2003. "An optimal threshold control for a BMAP/SM/1 queueing system with MAP disaster flow". *Automation and Remote Control* 64, No.9, 1442 - 1454.

Semenova, O.V. 2004. "Optimal control for a BMAP/SM/1 queue with MAP-input of disasters and two operation modes". *RAIRO - Operations Research* (accepted).

Tijms, H. 1976. "On the optimality of a switch-over with exponential controlling the queue size in a M/G/1 queue with variable service rate". *Lect. Notes Comput. Sci.* 40, 736-742.

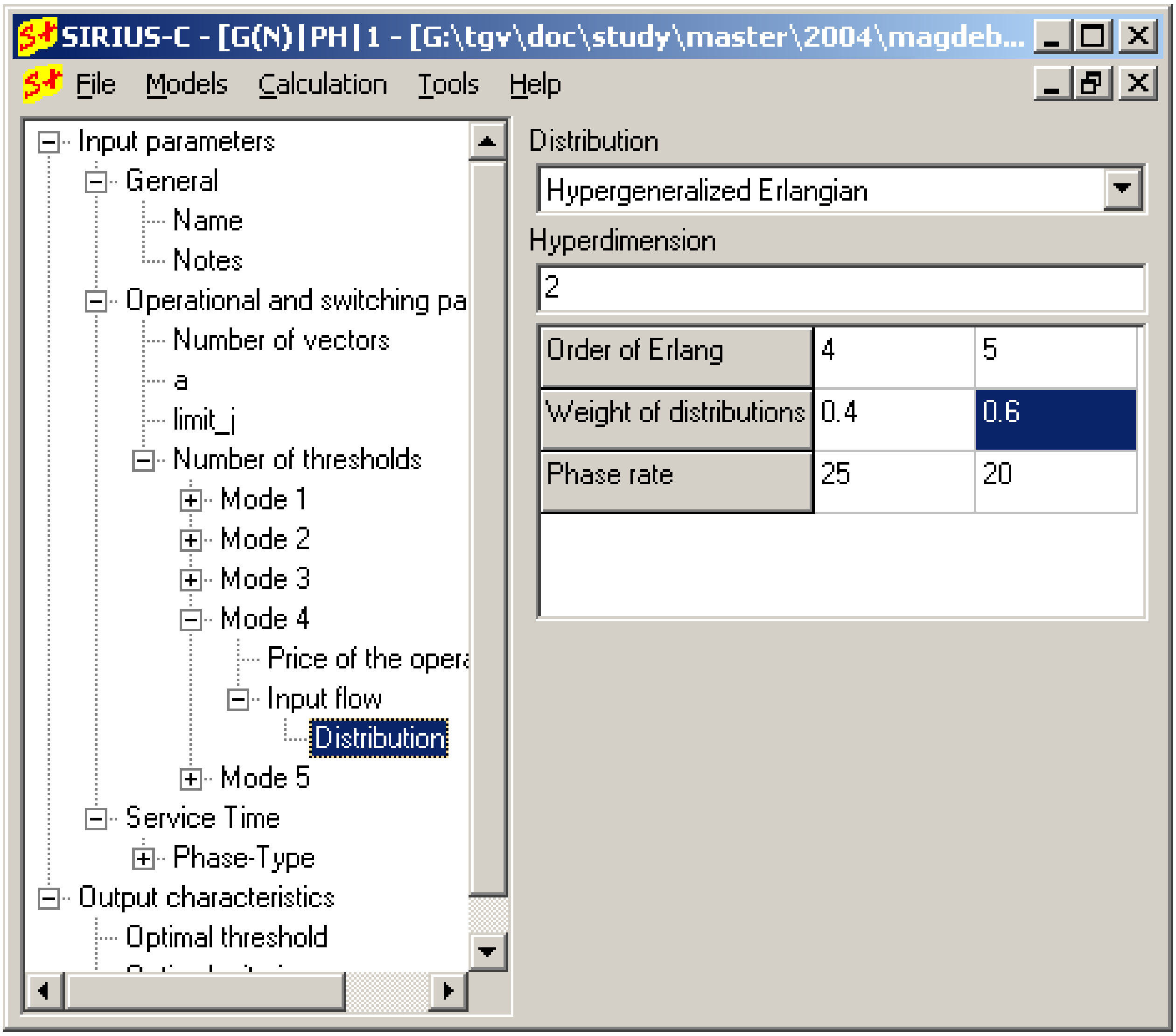

Figure 2: Screen Shot of Data Input for One of Operating Mode in Numerical Example

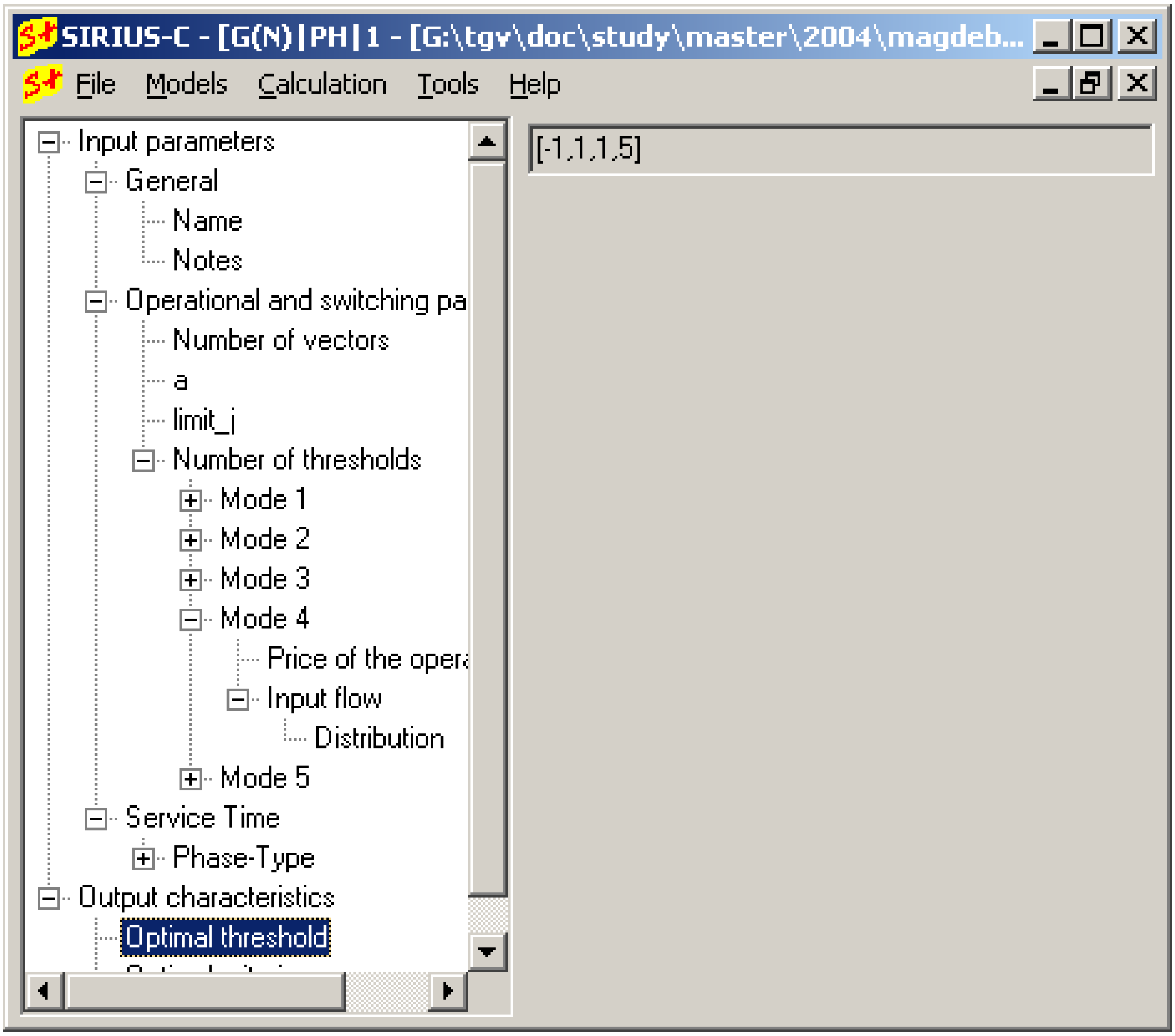

Figure 3: Screen Shot of Results With Thresholds Optimal Set